Adding \& Subtracting Mixed Numbers
Developed by Kristin Ulrich

## (Carrying and Borrowing)

Use a concrete model to explicitly show students how to carry and borrow when solving specific fraction addition and subtraction problems.


40-50 minutes.

## Objectives

Students will...

- Be able to solve fraction addition and subtraction problems.
- Be able to use concrete problems to solve fraction addition and subtraction problems.
- Be able to find the least common denominator (LCD) of two unlike fractions or mixed numbers.


## Common Core State Standards

CCSS.Math.Content.4.NF.B.3.C - Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
CCSS.Math.Content.5.NF.A. 1 - Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, $2 / 3+5 / 4=8 / 12+15 / 12=23 / 12$. (In general, $a / b+c / d=(a d+b c) / b d$.)

## Teacher Notes

This lesson is designed for students who have had some experience with adding and subtracting fractions with both like and unlike denominators, finding the Least Common Denominator (LCD), and adding and subtracting mixed numbers. You'll only be working with the yellow hexagons, the red trapezoids, the blue rhombuses, and the green triangles for this lesson. Below are the values for each shape:

## Hexagon = 1

Trapezoid $=1 / 2$
Rhombus $=1 / 3$
Triangle $=1 / 6$
These values are used based on the idea that a hexagon equals one whole. It takes two trapezoids to completely cover the hexagon, giving it the value of $1 / 2$. It takes three rhombuses to cover the area of a hexagon, giving it the value of $1 / 3$. It takes six triangles to cover the area of a hexagon, giving it the value of $1 / 6$. Explain this concept to students as you work through the introduction and activity with them.

## Introduction $\lll \lll \lll \lll \lll \lll \lll \lll \lll \lll \lll \lll \lll \lll \lll \lll \lll \lll \lll \lll<$

In this part, students should have one of their yellow hexagons in front of them. You will then be guiding the students through placing various shapes on top of the hexagon. This will help them see how many of each shape fully cover the hexagon, as well as the different groups of fractions that make up each whole hexagon (3 rhombus shapes $=1$ hexagon). This exercise familiarizes students with the values of each shape.

1. How many red trapezoids does it take to fully cover the hexagon? (2 trapezoids)
2. If one trapezoid is placed over the hexagon, what fraction of the hexagon is covered? ( $1 / 2$ )
3. If two trapezoids are placed over the hexagon, what fraction of the hexagon is covered? ( $2 / 2$ or 1 whole)
4. How many blue rhombuses does it take to fully cover the hexagon? (3 rhombuses)
5. If one rhombus is placed over the hexagon, what fraction of the hexagon is covered? ( $1 / 3$ )
6. If two rhombuses are placed over the hexagon, what fraction of the hexagon is covered? (2/3)
7. If three rhombuses are placed over the hexagon, what fraction of the hexagon is covered? ( $3 / 3$ or 1 whole)
8. How many green triangles does it take to fully cover the hexagon? (6 triangles)
9. If one triangle is placed over the hexagon, what fraction of the hexagon is covered? ( $1 / 6$ )
10. Continue in this same fashion with the rest of the triangle pieces until all six pieces are covering the hexagon, making the fraction $6 / 6$ or 1 whole.

## Activity



1. Have students look at problem 1 and determine what pattern blocks would be used to represent $15 / 6$ ( 1 yellow hexagon and 5 green triangles). Students should then draw that representation in Box 1 with an addition sign after it.
2. Students should now determine what pattern blocks represent $11 / 3$, then draw that representation in Box 2 (1 yellow hexagon and 1 blue rhombus).
3. Since the fractions have different denominators, students should be able to say that the next thing they need to do is make the denominators the same. To do this, they should first determine which denominator is the LCD of 3 and 6 (6). Ask which color/shape of their pattern blocks represents sixths (green triangles). Since one of the addends is already in sixths, they will only need to change $11 / 3$ to sixths. Ask how many green triangles it takes to cover the blue rhombus (2). This means that $12 / 6$ is the same as $11 / 3$. Have students cross off the blue rhombus in Box 2 and replace it with two green triangles.
4. Students should know that the next step is to add the fractions of $5 / 6$ and $2 / 6$ together. They should be able to come up with $7 / 6$. Now they can add up the whole numbers, represented by the yellow hexagons. They should have two yellow hexagons. Putting the fraction answer and the whole number together, students should come up with $27 / 6$ as the answer to the problem. Have students draw that answer in Box 3 .
5. Students should already be aware that this is not the final answer because the final answer can't include an improper fraction. Ask students how many green triangle sixths it takes to make one yellow hexagon (6). Have them exchange six of the triangles for a hexagon. Now they should have three yellow hexagons and one green triangle, which gives them $31 / 6$. That final answer should be drawn in Box 4 .
6. Move on to problem 2. Students should select three hexagons and one trapezoid to represent $31 / 2$, then draw that in Box 1 with a subtraction sign after it.
7. Students should select a hexagon and two rhombuses to represent $12 / 3$, then draw that in Box 2.
8. Students should now know that they need to figure out the LCD of 2 and 3 (6), which will tell them they need to use triangles. It will take three triangles to cover the trapezoid in $31 / 2$, demonstrating that $1 / 2$ is the same as $3 / 6$. Students should cross off the trapezoid in Box 1 and replace it with three triangles. They should also see how many triangles it takes to cover the two rhombuses in $12 / 3$ (4), demonstrating that $2 / 3$ is the same as $4 / 6$. They should cross off the two rhombuses in Box 2 and replace them with four triangles.
9. Students should now subtract the fractions from each other. Since they are working with $3 / 6-4 / 6$, they are going to need to borrow, just like they would if they were subtracting whole numbers. Students should trade in one of the hexagons in $31 / 2$ for six triangles, then draw that representation in Box 3.
10. Since they have traded in a hexagon for triangles, the problem students are trying to solve now reads $29 / 6-14 / 6$. As before, students will begin by subtracting the fractions from each other, which they can now do, then subtract the whole numbers. The final answer is $15 / 6$.


## Activity Cont.

11. Since the first addend of problem 3 is a whole number, students will only need hexagons to represent the number. They should select four hexagons, then draw them in Box 1 along with a subtraction sign.
12. For Box 2 , students will need to select and draw one hexagon and one rhombus to represent $11 / 3$.
13. See if students can figure out what they should do next. Hopefully they will know they need to exchange a hexagon for three rhombuses, but if they are struggling, prompt them to go back and look at what they did in Box 3 of the previous problem. If they do that and think that they need to exchange a hexagon for six triangles, ask them what the denominator is in the second number to help them get back on track. They should then draw the correct exchange in Box 3 .
14. The problem should now be $33 / 3-11 / 3$. They will subtract the fractions first, then the whole numbers, to come up with $22 / 3$. They should draw the answer in Box 4.
15. Have students work on problem 4 on their own. Before letting them do so, remind them that the first number goes in Box 1 , the second number goes in Box 2, any borrowing or carrying should be done in Box 3, and the final answer goes in Box 4. Be sure they are using their pattern blocks to solve the problem.

## Checking for Understanding

After students have completed problem 4, check for understanding by using the following line of questioning:

1. What did you draw in Box 1 ? (3 hexagons)
2. Why is there only one type of pattern block? (It's a whole number without a fraction.)
3. What did you draw in Box 2 ? ( 2 hexagons and 5 triangles)
4. Can you start subtracting there? (No)
5. Why not? (There is nothing to take the $5 / 6$ away from.)
6. What did you need to do before you subtract? (Borrow)
7. How do you borrow with mixed numbers? (Take from the whole number.)
8. What did you need to exchange one of the three hexagons for? (6 triangles)
9. How did you draw that in Box 3 ? ( 2 hexagons and 6 triangles minus 2 hexagons and 5 triangles)
10. What did you do next? (Subtract the triangles. That leaves 1 triangle or $1 / 6$. .)
11. What comes next? (Subtract the hexagons. That leaves no hexagons.)
12. What is the final answer? ( 1 triangle or $1 / 6$, which should be drawn in Box 4.)

## $\Delta$ Practice



Have students complete the rest of the problems. Point out that two of the problems are addition and two are subtraction, so they need to make sure they are using the correct operation. Each problem will require them to either carry or borrow, so they should look back at the problems the class did together if they get stuck. Every box on the worksheet should be filled in when they are finished.

## Intervention

- Focus on either just the addition or just the subtraction problems. It may help with mastery if students only have one skill to focus on at a time. Here are a few additional addition or subtraction problems if you choose to isolate the skills:

Addition<br>$15 / 6+11 / 6$<br>$31 / 2+22 / 3$ $12 / 3+33 / 6$

Subtraction
$21 / 6-11 / 3$
$31 / 2-15 / 6$


## Extension

- Have students create two more addition problems of their own that require carrying. They should create their own boxes on the back of the paper to show how they would represent those problems with pattern blocks.
- Have students create two more subtraction problems that require borrowing. They should draw they problems they came up with on boxes on the back to show which pattern blocks they would use to represent the problem.

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