Hands-On Math Geometry

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Contents

To the Teacher

Since the early 1970s, we have been teaching math to learners of all ages, from young children to adults, who represent many different cultures and socioeconomic backgrounds. We believe that all learners *can* do math by first overcoming any math anxiety and then by participating in meaningful, cooperative learning activities that relate to various learning modalities (e.g., auditory, visual, kinesthetic, and tactile) of each learner.

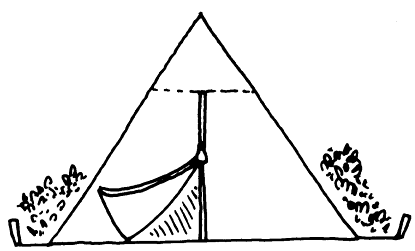
With the release of the NCTM Standards in 1988 and in 2000 the NCTM Principles and Standards for School Mathematics, teaching mathematics has become more student-driven and hands-on. Emphasis on deductive and inductive reasoning through a discovery process enables a student to truly *understand* mathematics. We feel the labs presented in this book address these concerns. First, our labs address students' various learning styles. We provide hands-on activities of measuring, constructions, etc., which address the kinesthetic learner. We give opportunities to write and communicate ideas and to visualize concepts, thus including the visual learner. Finally, we furnish opportunities for group discussions and talking through problems, enabling the auditory student to be involved.

The study of Euclidean geometry lends itself to discovery of theorems through hands-on applications. Students who derive their own meaning for various theorems will own them and will understand what the theorems mean.

We hope you enjoy trying these activities with your students. We believe learning should be learner-centered, not teacher-driven. As quoted in the NCTM Principles and Standards, "students should conduct . . . explorations which will allow them to develop a deeper understanding of important geometric ideas. . . ."

—Pam and Judy

Triangles



Activity 14

Similar Triangles

Teacher Page

Learning Outcomes

Students will be able to

- solve for unknown heights using proportions.
- measure various lengths with appropriate measuring tools.

Overview

Student groups will calculate the height of buildings, flagpoles, or trees by measuring the objects' shadows and solving a proportion formula. Students will also calculate each other's heights using the same method.

Time Requirements

30 minutes

Group Size

Pairs or groups of three

Materials

- rulers
- tape measures
- lab sheets
- calculators

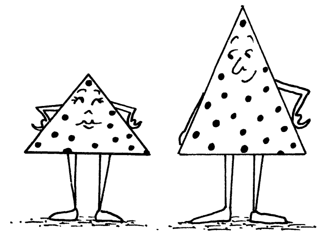
Procedure

1. This lab works best outdoors during a sunny day. If you don't have these conditions, then you could use a lamp with a bright light to cast shadows.
2. Sometimes building shadows may be hard to measure. Students must measure the perpendicular distance of the shadow. If your site has trees or flagpoles, they may work better.
3. When students measure their heights, the calculated heights may not match their exact heights. Discuss why this may happen. Some students may question where to measure the shadow, from the back of the student's foot or in the front. Have students experiment to see which gives the more accurate measure. Make sure the students set up their proportions correctly and in the correct order.

Name________________________ Date ____________

ACTIVITY 14

Similar Triangles



Part One

In this lab, we are going to try to figure out the height of a building or flagpole using the process of similar figures and proportions.

1. Go outside and measure the shadow of a building or flagpole in feet.

   Measure of shadow of building or flagpole: ____________________

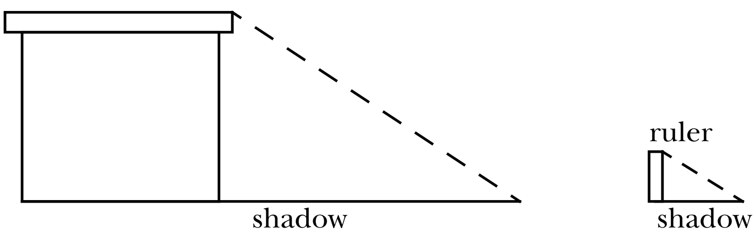
2. Now place a ruler perpendicular to the ground and measure its shadow.

   Measure of shadow of ruler in inches: ____________________

3. Change the measure of the ruler's shadow to feet by dividing by 12.

   Measure of ruler's shadow in feet: ____________________

4. You are now ready to calculate the height of the building or flagpole.



Proportion: $\dfrac{\text{Height of building}}{\text{Length of building's shadow}} = \dfrac{\text{Height of ruler}}{\text{Length of ruler's shadow}}$

Fill in with your measurements: $\dfrac{\text{Height of building \_\_\_\_\_\_}}{\text{Length of shadow \_\_\_\_\_\_}} = \dfrac{\text{Height of ruler \_\_\_\_\_\_}}{\text{Length of shadow \_\_\_\_\_\_}}$

What is your calculated building height? ____________________

Part Two

1. Have your partner measure the length of your shadow and try to calculate your height.
2. Use the proportion of the length of the ruler to its shadow as the other part of the proportion. Does your calculated height come close to your actual height? ________

Activity 15

Pythagorean Theorem

Teacher Page

Learning Outcome

Students will be able to apply the Pythagorean theorem to real-life situations.

Overview

Students will use a variety of hands-on methods to learn about, test, and calculate with the Pythagorean theorem.

Time Requirements

40–60 minutes

Group Size

Pairs

Materials

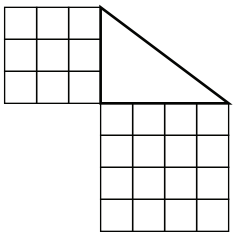
For each group

- 25 one-inch-square tiles
- 3" × 4" × 5" right triangles
- 26-inch piece of string
- centimeter rulers
- centimeter dot-matrix paper

Procedure

Part One

1. Pass out a 3" × 4" × 5" right triangle and 25 one-inch square tiles
2. Have groups build squares on each of the legs of the right triangle. If they do it correctly, all 25 tiles will be used, with 9 on the 3-inch side and 16 on the 4-inch side.

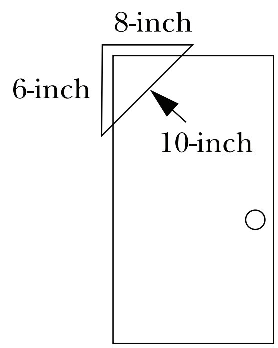


3. Have groups take the tiles and build a “square” on the hypotenuse using as many of the 25 tiles as they need. They will see that all the tiles will be used on the hypotenuse. This should show students that the sum of the squares of the legs equals the square formed on the hypotenuse.

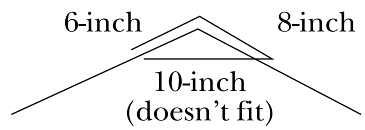
Part Two

1. In this activity, the students are using the converse of the Pythagorean theorem. That is, if the squares of two sides of a triangle equal the square of the hypotenuse, then the triangle must be a right triangle.
2. Using the 6-8-10-inch string, the groups will locate right triangles in the room and use the string to prove the triangles are right triangles. To do this, students will put the

6-inch length on one leg of the triangle and the 8-inch length on the other leg. The remaining 10-inch length should form the hypotenuse.



3. The groups should also try an object that doesn't have a right angle to show that the three pieces will not fit together.

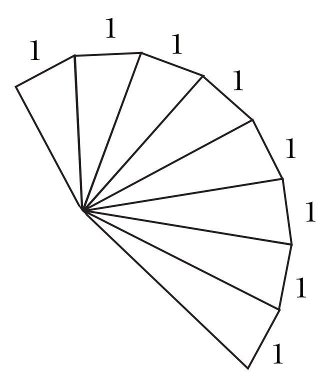


Part Three

1. Using dot-matrix paper students make several right triangles.
2. They then measure the legs and put the measurements into the Pythagorean formula to determine the length of the hypotenuse.
3. Teams then measure the hypotenuse with a ruler to verify the results. This activity gives students practice with the Pythagorean theorem with visual results to verify their work.

Part Four

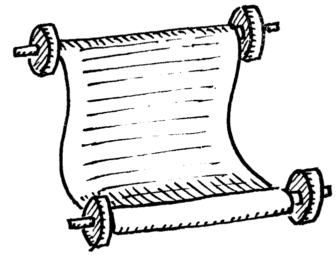
In this activity, the student will begin to see a spiral form. The key is to keep one of the legs constant with a length of one. The other leg is the hypotenuse of the previous right triangle.



Name____ Date ____________________

Activity 15

Pythagorean Theorem



Part One

Your teacher will give you a picture of a right triangle and some 1-inch tiles. Count out 25 tiles. Determine how long the **"legs"** (the sides of the triangle that form a right angle) are in tiles.

One leg is __________ tiles long.

The other leg is __________ tiles long.

Now form squares on each of these legs with the tiles. ***Example:*** If the leg were 6 tiles long, the square would be 6 tiles by 6 tiles. Fill in this square with the remaining tiles. If you do this correctly, you will use all 25 tiles.

Now look at the "slanted side" of the triangle. The slanted side is called the ***hypotenuse.*** What do you *estimate* the length of the hypotenuse to be in tiles?

Estimated length is __________ tiles.

Now use your tiles to see how long the hypotenuse is.

The hypotenuse is __________ tiles long.

Following the same procedure as above, use as many tiles as you need to make a square on the hypotenuse and fill it in with the tiles.

What do you notice?

____

____

Can you make a rule about the legs and hypotenuse of a right triangle from what you have observed? Please state it below.

____

____

The **Pythagorean theorem** states that if a triangle is a right triangle, then:

$$a^2 + b^2 = c^2$$

Explain in words what this means:

____

____

(continued)

Name______________________________ Date ______________

Activity 15

Pythagorean Theorem *(continued)*

Part Two

The **converse** of the Pythagorean theorem states that if a triangle has sides of length *a, b,* and *c,* and $a^2+b^2=c^2$, then the triangle is a right triangle. This means that you can determine whether a triangle has a right angle by testing the sides in the formula.

The converse is used in laying pipe, constructing houses, or anything that requires formation of a right angle.

Take a 26" piece of string, measure 6 inches, and place a knot at the 6" mark. From that knot, measure 8 inches and knot again. The remaining section should be 10", so cut off any leftover string.

Could 6, 8, and 10 be the measures of the sides of a right triangle? ______________

How do you know? ____

____

Using this string, find places around the room that appear to form right angles and use this measuring device to check them. List below the angles you examined, and whether they were right angles.

____

____

____

Now find an angle that is not a right angle, and test to see that the 6-8-10 string will not work.

(continued)

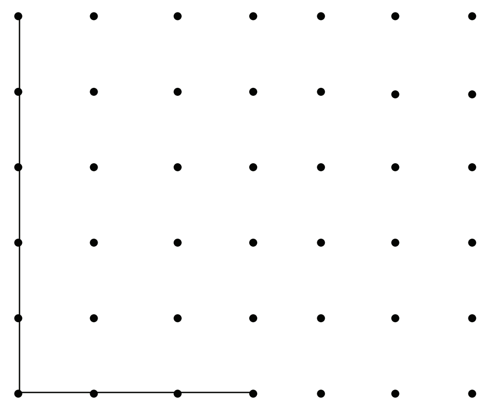
Name________________________ Date ________________

ACTIVITY 15

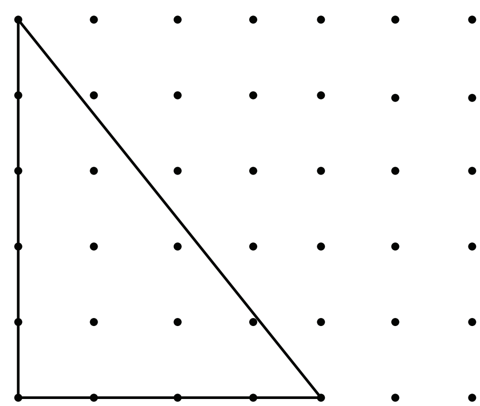
Pythagorean Theorem *(continued)*

Part Three

On your matrix paper, draw a right triangle by connecting dots up and down or right and left. The legs can be any length you want.



Then connect the diagonal (the hypotenuse).



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Name______________________________ Date ______________

ACTIVITY 15

Pythagorean Theorem *(continued)*

Test the lengths with the Pythagorean theorem. In this example, the triangle drawn has the lengths of 4 and 5 cm. Plug these lengths into the Pythagorean formula to find the length of the hypotenuse.

$a^2 + b^2 = c^2$

as $4^2 + 5^2 = c^2$

$16 + 25 = c^2$

$41 = c^2$

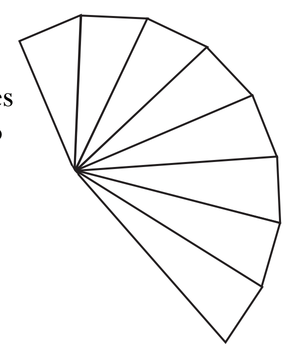
$\sqrt{41} = \sqrt{c^2}$

$6.4 = c$

Now measure the hypotenuse with your centimeter ruler to verify this solution.

Try several examples on the dot-matrix paper to demonstrate the Pythagorean theorem.

Part Four



In nature, the spiral shell is formed by connected right triangles in which one leg stays one unit long, while the other leg increases to the length of the previous hypotenuse.

Look at the diagram on the right. Try creating this spiral on your grid paper.

See if you or any of your classmates can find a shell, or a photograph of a shell, that illustrates this.

Activity 16

Median Line of a Triangle

Teacher Page

Learning Outcomes

Students will be able to

- determine the midpoints of two sides of a triangle using a compass.
- discover that a segment whose endpoints are the midpoints of two sides of a triangle is parallel to the third side of the triangle, and its length is one half the length of the third side.

Overview

Through construction and measurement, students will discover properties of the line that connects the midpoints of two sides of a triangle.

Time Requirements

30–45 minutes

Group Size

Pairs

Materials

- compass
- protractor
- lab sheet
- ruler

Procedure

Before the lab, review the use of a compass with the class. Have them practice drawing circles first. Then introduce/review how you manipulate the compass to bisect line segments.

You may also need to review how to use a protractor to record angle measurements.

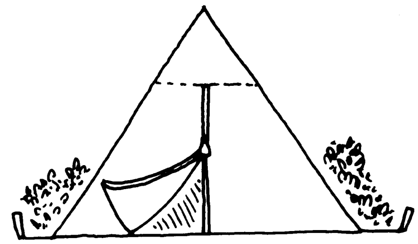
1. After reviewing how to bisect a line, have the students bisect line segments $\overline{MN}$ and $\overline{MP}$.
2. Next have the students connect the two midpoints found and identify these points as S and T, respectively.
3. When they measure $\angle MST$ and $\angle N$, they will find that they are equal.
4. The same will occur with $\angle P$ and $\angle MTS$. They will be equal.

Teacher Page

5. Line segments $\overline{ST}$ and $\overline{NP}$ are parallel because if corresponding angles on the same side of a transversal are equal, the lines are parallel.
6. The students should notice that the measure of $\overline{ST}$ is half the measure of $\overline{NP}$.
7. Students will discover that this will hold true for any triangle regardless of the shape.

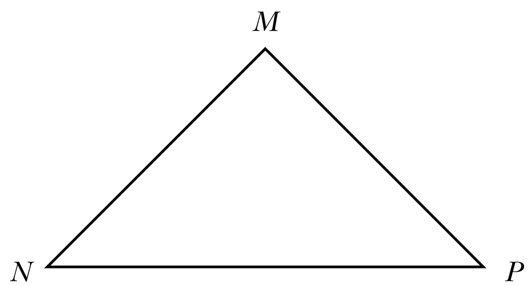
Name________________________________ Date ____________________

ACTIVITY 16



Median Line of a Triangle

1. Use a compass to bisect line segments $\overline{MN}$ and $\overline{MP}$.
2. Label the midpoint of $\overline{MN}$ as S and the midpoint of $\overline{MP}$ as T, then draw line segment $\overline{ST}$.
3. Use a protractor and measure $\angle MST$ and $\angle N$. $\angle MST$ = ________________
   $\angle N$ = ________________
4. Next measure $\angle P$ and $\angle MTS$. $\angle P$ = _______ $\angle MTS$ _________



5. Compare the angle measures. What relationship does this suggest for line segments $\overline{ST}$ and $\overline{NP}$? ____

   ____

   ____

   Why? ____

   ____

6. Measure $\overline{ST}$ and $\overline{NP}$. How do these values compare?

   ____

   ____

   ____

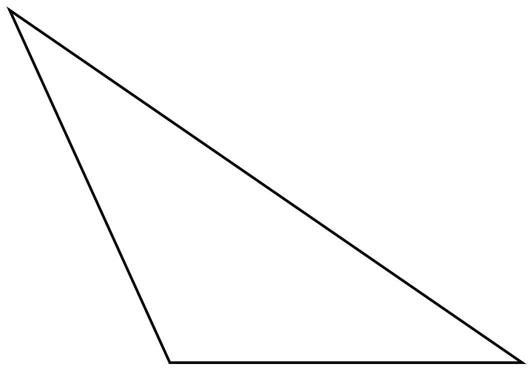
(continued)

Name________________________________ Date ______________

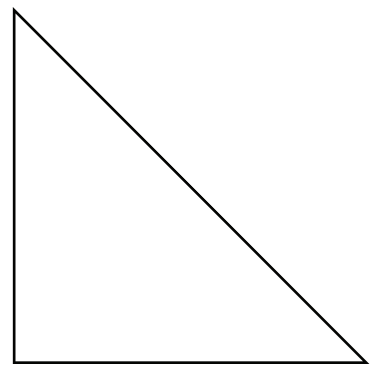
Activity 16

Median Line of a Triangle *(continued)*

7. Repeat the above steps for the obtuse and right triangles below.



Obtuse triangle



Right triangle

Do you get the same results? Explain.

____

____

____

____