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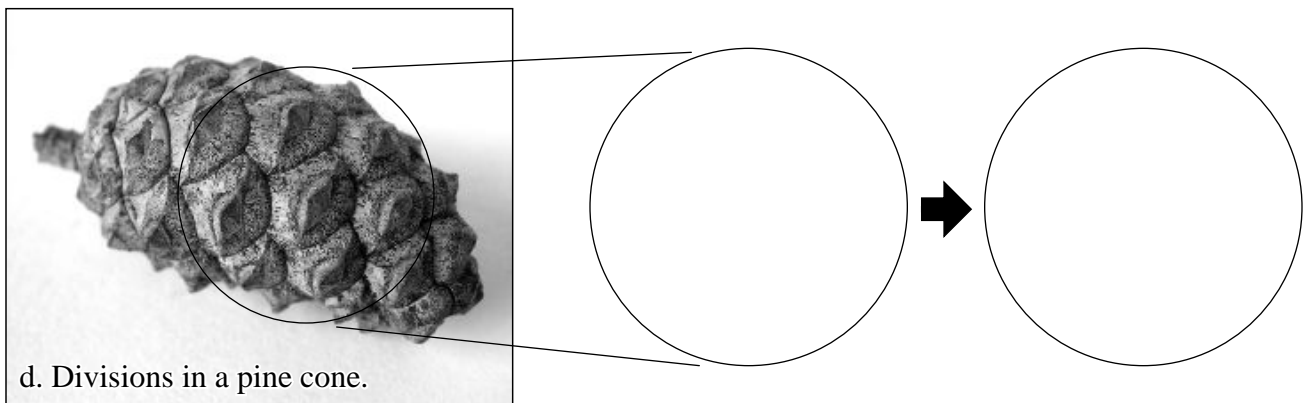
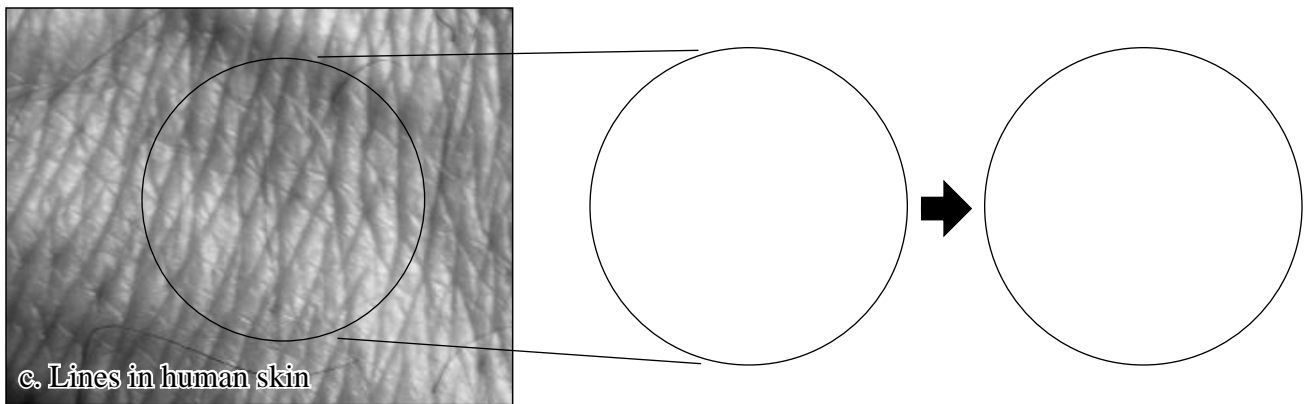
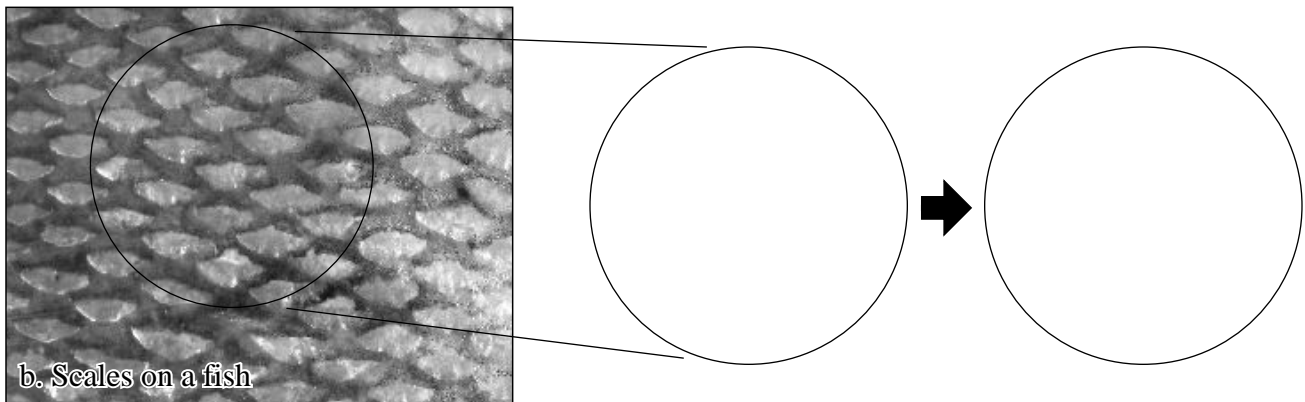
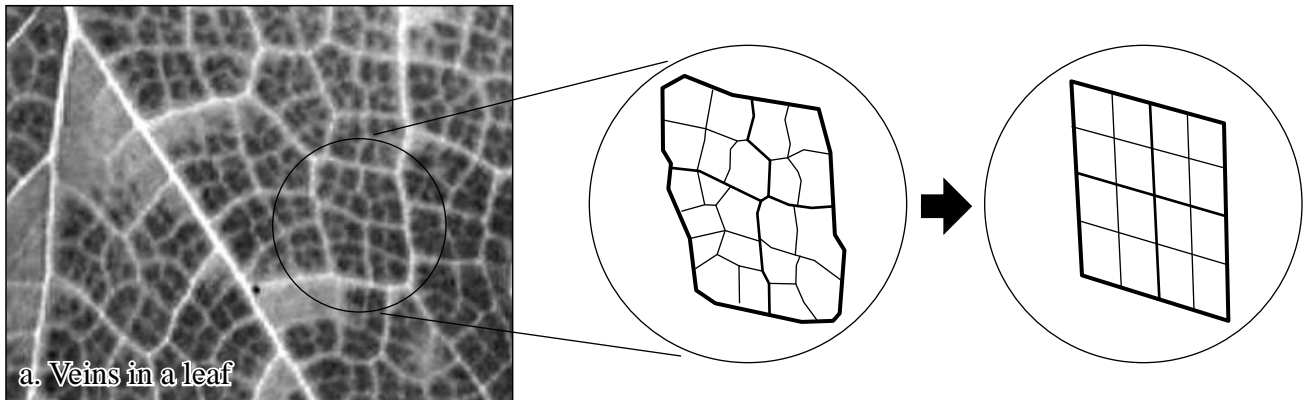
Bibliography

Glossary

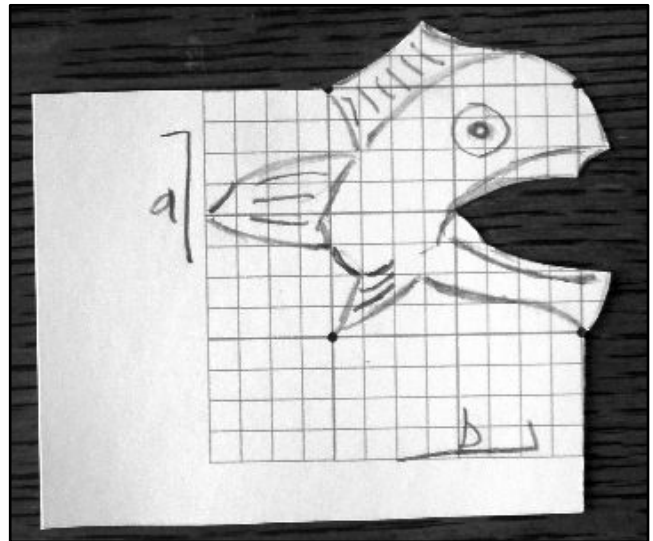
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## Worksheet 2-2. Modeling natural tessellations using geometric tessellations

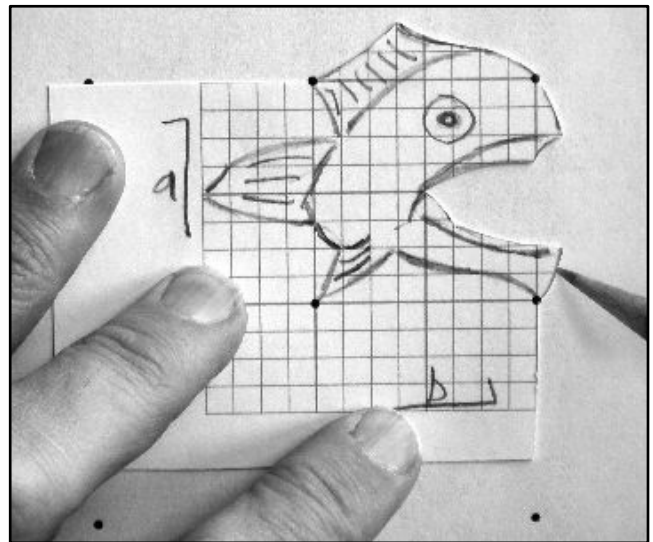
For each real-world object, first draw a portion as a tessellation of tiles that are close to the photograph in shape, and then as a more geometrically-regular tessellation, as shown in the example.



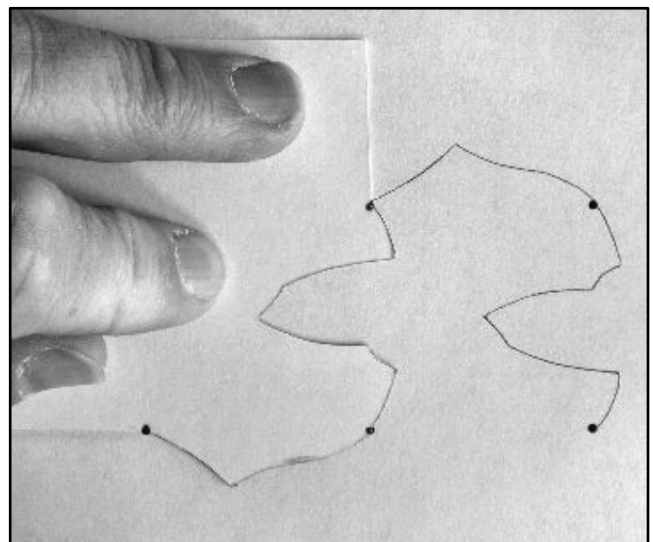
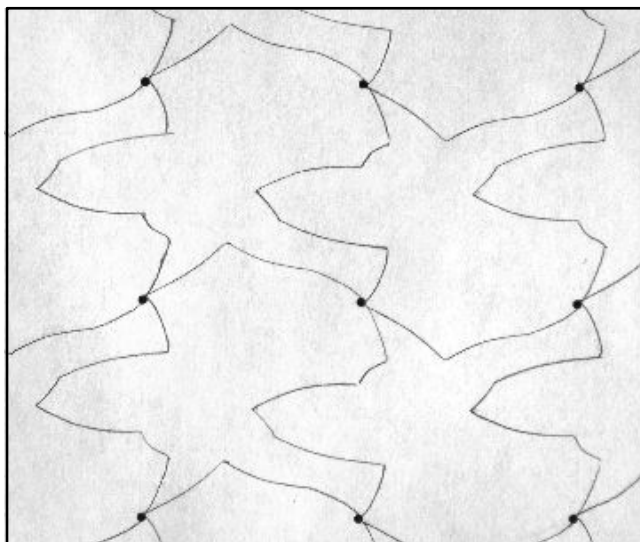
*Step 6.* Once you are satisfied with your tile, carefully cut one curve of each type. In the example here, cut one of the two “a” curves, and one of the two “b” curves. Cut off the bulk of the sheet, so you have a piece similar to that shown at top, right, that can be used as a tracing template.



*Step 7.* Use this template to draw a tile on the Square Tessellation Grid sheet. Carefully align the shape to the grid using the dots. Hold it firmly in place and tightly trace the cut curves, as shown at middle right. It’s important to use a sharp pencil and trace right at the edge of the template. Then reposition the template to trace the other curves. The template page shows how the tiles fit together, and how each tile is oriented. Use it as a guide to ensure the curves are traced in the correct orientations. Template 7-3 shows that both the “a” and “b” curves are oriented the same within a column, but that they are mirrored from one column to the adjacent column. This means the template needs to be flipped over to draw these curves, as shown at bottom, right.



*Step 8.* Repeat this process of aligning the template with dots and tracing the template until the entire Square Tessellation Grid sheet (or as much as you want) is covered with your tessellated shapes.

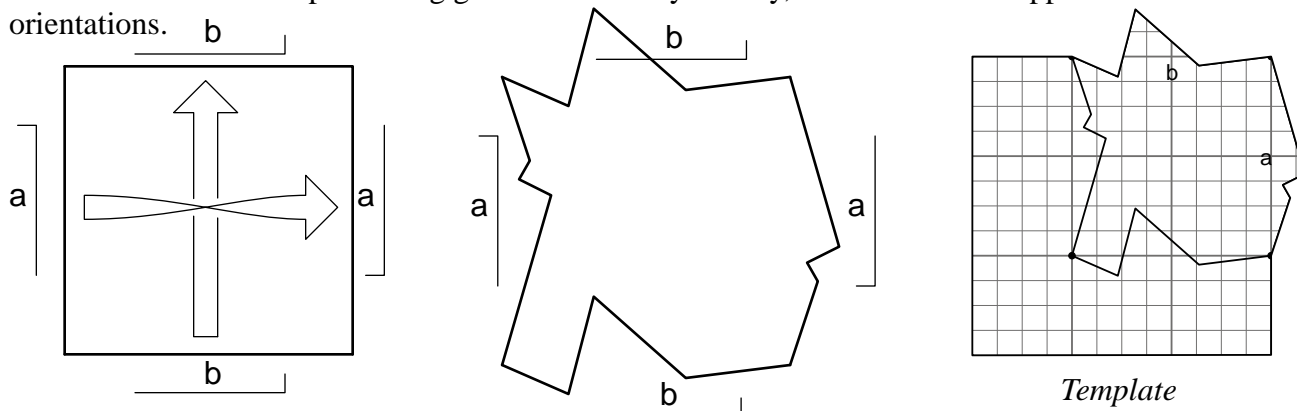


## Template 7-3

Symmetry group pg, Heesch type TGTG, Escher's notebook nos. 108 and 109

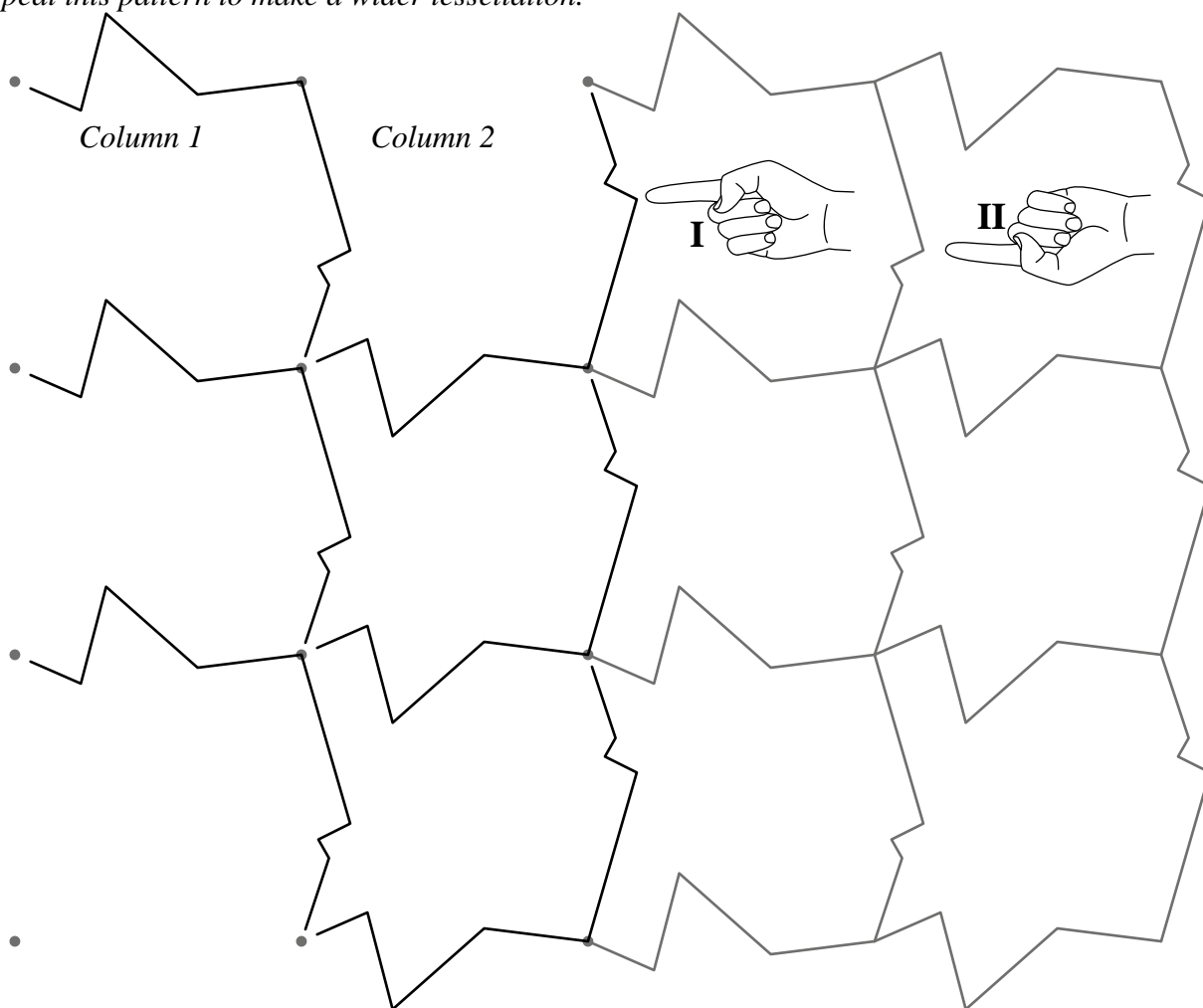
### Tessellation with glide reflection symmetry

One of two independent line segments simply translates from one edge of the square to the opposite edge, while the other line segment is reflected from one side of the square to the other. This results in a tessellation possessing glide reflection symmetry, in which the tile appears in two distinct orientations.



To draw this tessellation using the grids:

Cut along the top and right edges of your tile. Form Column 1 of lines as indicated below by tracing the cut edges. Then flip your template over top to bottom and form an adjacent Column 2 of lines. Repeat this pattern to make a wider tessellation.



## Activities

The steps below are described in more detail in the pages on creating a tessellation by hand (pp. 72-74). Unless otherwise noted, a pencil should be used. In a classroom setting, the teacher may wish to demonstrate the steps before having the students perform them.

### Activity 7-1. Creating a tessellation with translational symmetry

*Materials:* Copies of template sheet 7-1 on copier paper, copies of the Square Tile Grids sheet on card stock, copies of the Square Tessellation Grid sheet on a paper that can be drawn and colored on, scissors or craft knife, pencil, pen, eraser, crayons or colored pencils (or other means of coloring a tessellation).

*Objective:* Learn to create, by hand, a tessellation that possesses translational symmetry.

*Vocabulary:* Translational symmetry, template, vector, tile, prototile, motif.

#### *Activity Sequence:*

1. Write the vocabulary words on the board and discuss the meaning of each.
2. Pass out the copies and other materials.
3. On the bottom portion of the template sheet, have the students draw two different vectors indicating translation distances and directions that would cause the tessellation to perfectly overlap itself.
4. Have the students mark the letters and flags for each of the four Square Tile Grids in a similar fashion to how they're marked on the template sheet.
5. Using the letters and flags as a guide, have them draw curves connecting the grid dots on at least one of the grids. The two "a" lines should be identical, as should the two "b" lines.
6. Have the students identify a motif in (one of) their shape(s). Then have them refine the shape and rough in interior details.
7. Have them further refine the shape and interior details to produce a prototile.
8. Have them cut out one curve of each type (one for each letter) and cut off the bulk of the sheet to create a template, as shown on the template sheet.
9. Have them use their templates to draw the outlines of the tiles on the Square Tessellation Grid sheet, as directed on the Template 7-1 sheet, continuing until the sheet is filled up.
10. Have them lightly sketch in key interior details for each tile.
11. Have the students use an ink pen to go over the tile outlines and interior details for the entire tessellation. After the ink is fully dry, unwanted pencil marks can be erased.
12. Have them color their tessellations. Before starting, you may wish to discuss coloring options.

#### *Discussion Questions:*

1. What motif did you use for your tile? Do you think it was effective? How would you change it if you could do it over again?
2. What step did you find the most challenging? Why?