Contents

Chapter 1

LINEAR EQUATIONS AND INEQUALITIES

Problems containing x to the first power

A proper and rigorous understanding of linear equations and their standard forms, linear segments and the associated algorithms, systems of multiple linear equations, and linear inequalities is an essential prerequisite for the study of calculus. Though the majority of calculus students are familiar with the topics in this chapter, mere familiarity is insufficient. In order to succeed in the more advanced topics of the chapters that follow, student mastery of these foundational skills and concepts must be ensured.

Points and lines are the most basic geometric concepts, so you'll need to understand how they are related before you can move on to more complex functions and their graphs. You'll need to know how to create equations of lines, graph lines in the coordinate plane, and even find the lengths and midpoints of line segments. You'll also need to know what to do with expressions containing $<$, $>$, $\le$, and $\ge$ signs. Once you've got that down pat, you'll review how to find solutions to systems of equations and inequalities (where you're working with more than one equation or inequality at a time).

Linear Geometry

Creating, graphing, and measuring lines and line segments

1.1 Solve the equation: $3x-(x-7)=4x-5$.

Distribute –1 through the parentheses and combine like terms.

$$3x-x+7=4x-5$$
$$2x+7=4x-5$$

Subtract $4x$ and 7 from both sides of the equation to separate the variable and constant terms.

$$\begin{array}{rrrrrrr} 2x & + & 7 & = & 4x & - & 5 \\ -4x & - & 7 & & -4x & - & 7 \\ \hline -2x & & & = & & - & 12 \end{array}$$

Divide both sides by –2 to get the solution.

$$\frac{-2x}{-2}=\frac{-12}{-2}$$
$$x=6$$

1.2 Calculate the slope, m, of the line $4x-3y=9$.

Slope-intercept form of a line is y = mx + b, where m is the slope of the line and b is the y-intercept.

Solve the equation for y in order to rewrite it in slope-intercept form.

$$-3y=-4x+9$$
$$y=\frac{4}{3}x-3$$

The slope of the line is the coefficient of x: $m=\frac{4}{3}$.

1.3 Prove that the slope of a line in standard form, $Ax+By=C$, is $-\frac{A}{B}$.

The equation's in standard form if it has: (1) No fractions, (2) Only x- and y-terms on the left side, (3) Just the constant on the right side, and (4) A positive x-coefficient.

Write the equation in slope-intercept form by solving it for y.

$$Ax+By=C$$
$$By=-Ax+C$$
$$y=-\frac{A}{B}x+\frac{C}{B}$$

The coefficient of x is the slope of the line: $m=-\frac{A}{B}$.

1.4 Rewrite the linear equation $3x-4\left(x-\frac{2}{3}y\right)=\frac{4}{5}x-(7y+3)$ in standard form.

Distribute the constants and combine like terms.

$$3x-4x+\frac{8}{3}y=\frac{4}{5}x-7y-3$$
$$-x+\frac{8}{3}y=\frac{4}{5}x-7y-3$$

Multiply by 15, the least common denominator, to eliminate fractions.

$$-15x + 40y = 12x - 105y - 45$$

Separate the variable and constant terms.

$$-27x + 145y = -45$$
$$27x - 145y = 45$$

Multiply the entire equation by −1 so that the x-coefficient is positive. (It's a requirement of standard form.)

1.5 Write the equation of the line passing through the points (–3,–8) and (–6,2) in slope-intercept form.

Calculate the slope of the line.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-(-8)}{-6-(-3)} = \frac{10}{-3} = -\frac{10}{3}$$

Think of the point (−3,−8) as (x_1,y_1) and (−6,2) as (x_2,y_2), so $x_1 = -3$, $y_1 = -8$, $x_2 = -6$, and $y_2 = 2$.

Substitute the slope into the slope-intercept formula ($y = mx + b$) for m, replace x and y using one of the coordinate pairs, and solve for b.

$$y = mx + b$$
$$-8 = -\frac{10}{3}(-3) + b$$
$$-8 = 10 + b$$
$$b = -18$$

Substitute m and b into the slope-intercept formula.

$$y = mx + b$$
$$y = -\frac{10}{3}x - 18$$

1.6 Calculate the x- and y-intercepts of $3x - 4y = -6$ and use them to graph the line.

To calculate the x-intercept, substitute 0 for y and solve for x. Similarly, substitute 0 for x to calculate the y-intercept.

$$3(0) - 4y = -6 \qquad 3x - 4(0) = -6$$
$$-4y = -6 \qquad 3x = -6$$
$$y = \frac{3}{2} \qquad x = -2$$

Therefore, the graph of $3x - 4y = -6$ intersects the x-axis at (–2,0) and the y-axis at $\left(0,\frac{3}{2}\right)$, as illustrated by Figure 1-1.

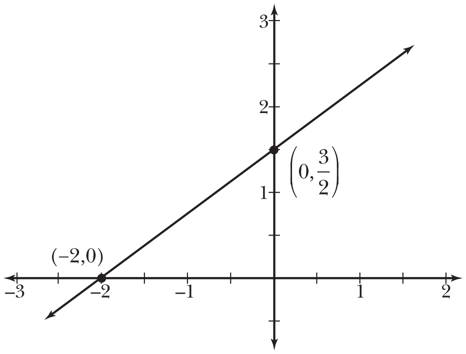

Figure 1-1

The graph of 3x – 4y = –6 with its x- and y-intercepts identified.

The point-slope formula creates an equation based on the slope of the line, m, and a point on the line, (x_1,y_1). Don't plug anything in for the x and y that don't have little numbers next to them.

1.7 Assume that line p contains the point (–3,1) and is parallel to $x - 4y = 1$. Write the equation of p in slope-intercept form.

Calculate the slope of $x - 4y = 1$ using the method of Problem 1.3.

$$m = -\frac{A}{B} = -\frac{1}{-4} = \frac{1}{4}$$

Plug this slope and the coordinates $(x_1,y_1) = (-3,1)$ into the point-slope formula.

$$y - y_1 = m(x - x_1)$$
$$y - 1 = \frac{1}{4}(x - (-3))$$
$$y - 1 = \frac{1}{4}x + \frac{3}{4}$$

Solve for y to express the equation in slope-intercept form.

$$y = \frac{1}{4}x + \frac{7}{4}$$

Note: Problems 1.8 - 1.10 refer to parallelogram ABCD in Figure 1-2.

1.8. According to a basic Euclidean geometry theorem, the diagonals of a parallelogram bisect each other. Demonstrate this theorem for parallelogram *ABCD*.

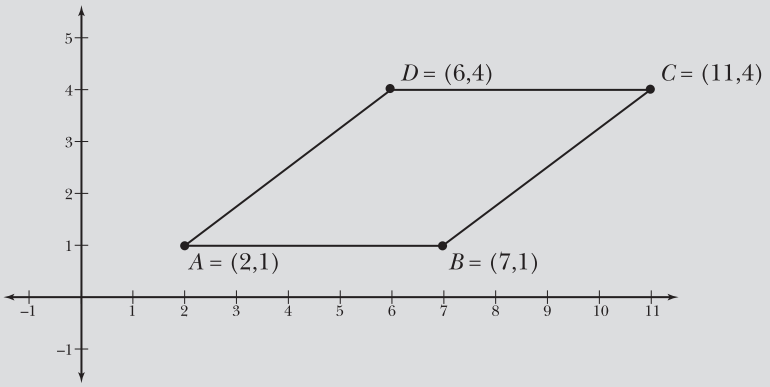

Figure 1-2

Parallelogram ABCD.

The midpoint of a line segment with endpoints (x_1,y_1) and (x_2,y_2) is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. In other words, the x-coordinate of a segment's midpoint is the average of the x-coordinates of its endpoints. The y-coordinate works the same way.

Calculate the midpoints of $\overline{AC}$ and $\overline{BD}$; the diagonals bisect one another if and only if those midpoints are equal.