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SECTION 1

Integers, Variables, and Expressions

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Teaching Notes 1.1: Using the Order of Operations

The order of operations is a set of rules for simplifying expressions that have two or more operations. A common mistake students make is to perform all operations in order from left to right, regardless of the proper order.

1. Present this problem to your students: $10 - 3 \times 2 + 2 \div 2$. Ask your students to solve. Some will apply the correct order of operations and find that the answer is 5, which is correct. Others will solve the problem in order from left to right and arrive at the answer of 8. Explain that this is the reason we use the order of operations. It provides rules to follow for solving problems.
2. Explain that to simplify an expression, the order of operations must be followed. State the following rules for the order of operations:
   - Perform all multiplication and division in order from left to right.
   - Perform all addition and subtraction in order from left to right.

   Emphasize that multiplication and division must be done first, no matter where these symbols appear in the expression.
3. Provide some examples, such as those below. Ask your students what steps they would follow to simplify the expressions. Then ask them to simplify each example.

$3 + 2 \times 5 \div 5$	Steps: $\times, \div, +$	Answer is 5.
$15 \div 5 \div 3 \times 2$	Steps: $\div, \div, \times$	Answer is 2.
$12 - 2 \div 2 \times 8$	Steps: $\div, \times, -$	Answer is 4.

4. Review the information and examples on the worksheet with your students.

EXTRA HELP:

Be sure you have performed all of the operations in their proper order.

ANSWER KEY:

(1) 5 **(2)** 21 **(3)** 4 **(4)** 18 **(5)** 24 **(6)** 188 **(7)** 1 **(8)** 39 **(9)** 8 **(10)** 48 **(11)** 10 **(12)** 16

(Challenge) The subtraction symbol should be inserted in the blank.

Name _____ Date _____

WORKSHEET 1.1: USING THE ORDER OF OPERATIONS

Mathematicians have agreed to simplify expressions that have no exponents or grouping symbols according to the following rules:

1. Multiply and divide in order from left to right.
2. Start at the left again and add and subtract from left to right.

EXAMPLES

$50 - 3 \times 2 \times 5 =$	$10 + 2 - 3 \times 4 =$	$32 \div 2 - 2 \times 3 =$
$50 - 6 \times 5 =$	$10 + 2 - 12 =$	$16 - 2 \times 3 =$
$50 - 30 =$	$12 - 12 =$	$16 - 6 =$
20	0	10

DIRECTIONS: Simplify each expression.

1. $12 - 2 \times 4 + 1$
2. $12 \times 4 \div 2 - 3$
3. $10 \times 2 - 2 \times 8$
4. $10 \times 2 - 6 \div 3$
5. $8 + 1 + 6 \times 5 \div 2$
6. $48 \div 2 \times 8 - 4$
7. $15 - 2 - 2 \times 6$
8. $35 + 8 - 12 \div 3$
9. $3 \times 7 - 8 - 5$
10. $20 \times 2 + 10 - 4 \div 2$
11. $8 - 4 + 2 \times 3$
12. $40 \div 8 - 5 + 3 \times 2 + 10$

CHALLENGE: Place the correct operation symbol in the blank so that $12 ___ 3 \times 2 + 2 = 8$.

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Teaching Notes 1.21: Finding Absolute Values of Expressions

To simplify an expression written within the absolute value symbol, students must think of the absolute value symbol as a grouping symbol. A common error is finding the absolute value of each number in the absolute value symbol and then finding the sum or difference.

1. Discuss the meaning of absolute value with your students. Point out that $|x|$ is the distance on the number line x is from zero. Depending on the abilities of your students, you may find it helpful to review 1.14: "Finding Absolute Values and Opposites."
2. Explain to your students that they should view absolute value as they would a grouping symbol, simplifying within it first before finding the absolute value of the number or numbers within it.
3. Offer the following examples:
   - $|3 - 12| = |3 + (-12)| = |-9| = 9$
   - $|3| - |12| = 3 - 12 = 3 + (-12) = -9$

   Emphasize that although the expressions look much alike, they are in fact quite different. In the first example, 3 and -12 are grouped within the absolute value symbol. In the second example, 3 and -12 are not grouped together but rather the difference of their absolute values must be found.
4. Review the examples on the worksheet with your students.

EXTRA HELP:

A negative symbol means the opposite. For example, $-|3|$ means the opposite of the absolute value of 3 which is -3.

ANSWER KEY:

(1) 11 **(2)** -18 **(3)** 28 **(4)** 41 **(5)** 84 **(6)** -31 **(7)** 48 **(8)** -42 **(9)** -5 **(10)** 23 **(11)** 10 **(12)** -10

(Challenge) 20 and 40.

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WORKSHEET 1.21: FINDING ABSOLUTE VALUES OF EXPRESSIONS

You can think of the absolute value symbol as a special type of grouping symbol. When working with expressions inside the absolute value symbol, you must find the absolute value of the simplified expression.

EXAMPLES

$\lvert 3 - 10\rvert =$	$\lvert 3(-6)\rvert =$	$\lvert -14 + 20\rvert =$	$3\lvert -15 + 10\rvert =$
$\lvert 3 + (-10)\rvert =$	$\lvert -18\rvert =$	$\lvert 6\rvert =$	$3\lvert -5\rvert =$
$\lvert -7\rvert =$	18	6	$3(5) =$
7			15

DIRECTIONS: Find each absolute value.

1. $\lvert 4 - 15\rvert$

2. $-3\lvert 6 - 12\rvert$

3. $\lvert (-7)(-4)\rvert$

4. $\lvert 15 + 26\rvert$

5. $7\lvert 3(-4)\rvert$

6. $-\lvert 15 - 46\rvert$

7. $\lvert 3(-4)\rvert + \lvert 2(-18)\rvert$

8. $-3\lvert 26 - 12\rvert$

9. $\lvert 5 - 4\rvert - \lvert -6\rvert$

10. $\lvert 3\rvert + 2\rvert - 10\rvert$

11. $\lvert -2 + (-4 \times 2)\rvert$

12. $-2\lvert 1 + 4\rvert$

CHALLENGE: Fill in the blank with a single number so that the following statement is true. There are two possible answers. Try to find both of them.
$\lvert 30 - ____\rvert = 10$

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Teaching Notes 1.22: Finding Square Roots of Square Numbers

When asked to find the square root of a number, some students make the mistake of dividing the number by 2. To remediate this problem, follow the procedure below.

SPECIAL MATERIALS:

Two sheets of graph paper for each student, rulers

1. Instruct your students to draw five squares: a 1×1 square, a 2×2 square, and so on up to a 5×5 square. Ask them to label two adjacent sides of each square and also write the area inside of each square. Ask them to write a number sentence to express the area of each.
2. Explain that the area of each square is equal to the length of a side squared. Thus, the area of a 3×3 square $= 9$ because $3^2 = 9$.
3. Suggest that students think of finding the square root of a number as finding the length of a side of a square. For example, $\sqrt{16} = 4$. 4 is the length of a side of a square that has an area of 16 units. Ask your students to find the square roots for each area they found (in step 1). Identifying these square roots will help them to memorize the process for finding square roots.
4. Review the list of square numbers on the worksheet with your students. Note that each of these numbers has a square root that is a positive integer. Also review the examples.

EXTRA HELP:

Square numbers are always positive.

ANSWER KEY:

(1) 3 **(2)** 6 **(3)** 10 **(4)** 9 **(5)** 1 **(6)** 2 **(7)** -5 **(8)** -1 **(9)** 11 **(10)** 10 **(11)** -3 **(12)** 2

(Challenge) $\sqrt{121} = 11$; $\sqrt{144} = 12$; $\sqrt{169} = 13$

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